Unit Code: CM20019 - S1 (Year 2006/07)
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Worksheet 1:
Terms, Substitutions, Unification, Propositional Logic
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Exercise 1
Find two different unifiers (not renamings) for the terms $f(X,Y)$ and $f(Y,a)$; show that one is more general then the other by applying the definitions.

Exercise 2
Let θ, η, γ be substitutions. Show that

$$(\theta \circ \eta) \circ \gamma = \theta \circ (\eta \circ \gamma).$$

Exercise 3
Prove by structural induction that each formula in the language of propositional logic without occurrences of connectives different than $\neg$, $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$ has an even number of parentheses, and that the number of $\wedge$ connectives is always less than or equal to the number of parentheses, when no parentheses are omitted.

Exercise 4
The rank $r(F)$ of a propositional formula is defined as follows:
$r(F) = 0$, if F is atomic,
$r(F) = max\{r(F_1), r(F_2)\} + 1$, if $F = (F_1 \cdot F_2)$ for any binary connective "$\cdot$",
$r(F) = r(F_1) + 1$, if $F = \neg F_1$.
Prove that $r(F)$ is less or equal to the number of occurrences of connectives in F.

Exercise 5
Consider the following formulae:
$(p \leftrightarrow q) \rightarrow (\neg q \wedge p)$ and $\neg(p \vee q) \rightarrow (\neg p \vee \neg q)$.

1. Build the truth tables for both of them;
2. Transform them in conjunctive normal form;
3. Transform them in disjunctive normal form;
4. Are the two given formulae tautologies?
5. Are there interpretations (and in this case say which ones) that make the formulae satisfiable?
6. Are there interpretations (and in this case say which ones) that make the formulae falsifiable?

Exercise 6
Statement: "If a formula F is satisfiable and G is falsifiable then $\neg F \leftrightarrow G$ is falsifiable". Is the statement true? Prove it or disprove it.

Exercise 7
Statement: " If $F \rightarrow G$ is satisfiable and F is satisfiable then G is satisfiable."
Is the statement true? Prove it or disprove it.

Exercise 8
Consider a set of propositional atomic formulae $\mathcal{F} = \{A_1, \ldots, A_n\}$, and the formula $F = (A_1 \rightarrow A_2) \wedge (A_2 \rightarrow A_3) \wedge \cdots \wedge (A_{n-1} \rightarrow A_n)$. Indicate under which assignments of truth values to A_i, the formula F is satisfiable.

Exercise 9
Prove that the following formulae are tautologies, using semantic concepts (i.e. the deduction theorem, the notions of interpretation and of logical consequence):
$(p \wedge (p \rightarrow q)) \rightarrow q$,
$(\neg p \vee q) \rightarrow (p \rightarrow q)$.

Exercise 10
In propositional classical logic, in how many different ways can you define a binary connective \$? And a n-ary connective $*$?

Exercise 11
This is a dialogue between two persons, A and B:
A: "If it rains, I'll take my umbrella. When I don't have it, I go by bus. Indeed, I always go by bus only when it rains."
B: "Whatever.... you should take your umbrella anyway".
Formalise A's opinion as sentences in propositional logic, in some language. Show, that B's answer is formally correct: it is a logical consequence of what A says.