CM 20019 - Coursework 1 - sample solution

EXERCISE 1:

1. Acceptable representations of Numbers in Morse code are finite strings of Digits, separated by one occurrence of the symbol #, if the string is made up by more than one Digit

Digits are 5-tuple of non alternating '•' and '—'.

A regular expression describing Numbers is

$$Digit^{+}\ (\#\ Digit)^{*}$$

2. Alphabet of terminal symbols: {'•', '—', '#'}
Alphabet of non-terminal symbols: {Digits, Numbers}

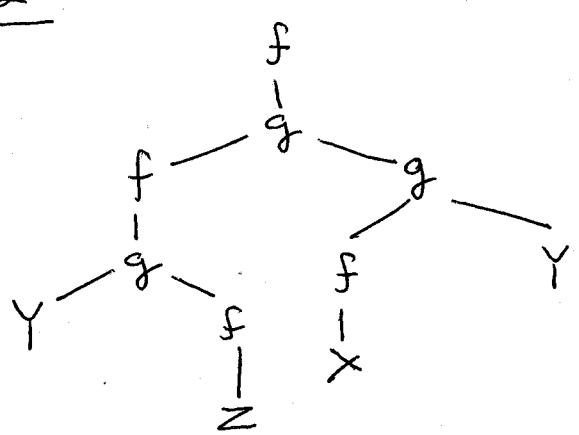
(hence, signature Σ is {'•', '—', '#', Digits, Numbers})

Initial symbol: Numbers

Production rules: Numbers ::= Digits | Digits # Numbers

Digits ::= •---- | ••--- | •••-- | ••••- | ••••• | -•••• | --••• | ---•• | ----• | -----

EXERCISE 2



EXERCISE 3:

1. $\sigma \circ \theta = [X := f(f(U,a),a),\ Y := U,\ U := f(f(U,a),a),\ V := f(a,U)]$

2. $\sigma \circ \theta = [X := g(X),\ Y := a,\ Z := g(b),\ U := X]$

EXERCISE 4

1. $S_1 = (\forall X)\ r(X,c) \rightarrow r(X,c)$
   (X: bound; X: free)

   $S_1\sigma_1 = (\forall X)\ r(X,c) \rightarrow r(f(a),c)$

2. $S_2 = (\forall X)\ r(X,Y) \rightarrow (\exists Y)\ r(Y,c)$
   (X: bound; Y: free; Y: bound)

   We rename those bound variables that have also free occurrences in S_2 (α-conversion) and get S_2':

   $S_2' = (\forall X)\ r(X,Y) \rightarrow (\exists Z)\ r(Z,c)$

   $S_2'\sigma_2 = (\forall X)\ r(X,a) \rightarrow (\exists Z)\ r(Z,c)$

3. $S_3 = (\forall X)(\exists Y)(r(X,Y) \rightarrow s(a,Y))$
   (X: bound; Y: bound; Y: bound)

   $S_3\sigma_3 = S_3$

4. $S_4 = (\forall X)(\exists Y)\ r(X,Y) \wedge s(a,Y)$
   (X: bound; Y: bound; Y: free)

   We rename those bound variables that have also free occurrences in S_4 (α-conversion) and get S_4':

   $S_4' = (\forall X)(\exists Z)\ r(X,Z) \wedge s(a,Y)$

   $S_4'\sigma_4 = (\forall X)(\exists Z)\ r(X,Z) \wedge s(a,a)$.

EXERCISE 5:

1. Not unifiable because of occur-check
2. $\theta = [X := Y] \circ [Y := a] = [X := a, Y := a]$
3. Not unifiable because of failure in attempt unifying two distinct function symbols (constants)

EXERCISE 6:

First order language with the following set A_R of predicate symbols (and arity), and empty set of function symbols A_F:

$$A_R = \{ f/1 , w/1 , a/1 , m/1 , d/1 \}$$

with intended meaning

$f^I(x)$ is true	iff	X is a fish
$w^I(x)$ is true	iff	X lives in water
$a^I(x)$ is true	iff	X is an animal
$m^I(x)$ is true	iff	X is a mammal
$d^I(x)$ is true	iff	X is a dolphin.

Formalization:

$$(\forall X)\ (\ f(X) \rightarrow a(X) \wedge w(X))$$
$$(\exists X)\ \ (a(X) \wedge m(X)$$
$$(\forall X)\ \ (d(X) \rightarrow m(X) \wedge w(X) \wedge \neg f(X))$$

EXERCISE 7:

Formalise in propositional logic with statements A, B:

"I'll make all exercises" = A

"I'll pass the exam" = B

The question is : $(A \rightarrow B) \stackrel{?}{\equiv} (\neg A \rightarrow \neg B)$

and this is false, because $(A \rightarrow B) \equiv (\neg B \rightarrow \neg A)$.