Unit Code: CM20019
Unit Lecturer: Dr. P. Bruscoli

Coursework 2 – due 16 November 2006

Please read the following information and instructions for the coursework submission process:

- Date, Time, Location for submission:
  This coursework is to be submitted through the coursework post box located outside 1W2.23 by 16 November, at 5:00 pm. Please be careful to select the box relative to this course.
- Form of submission:
  This coursework is to be submitted in written form, on paper, bound securely. Please make sure it contains information to identify you, clearly written: surname, first name, and student number. All students must include the coursework signature sheet.
- The proportion of the total assessment for this coursework:
  This is the second of three courseworks, and they all will contribute the 25% of the final grade. This specific coursework contributes 100 points (over a total of 300 points in three courseworks) and contributes for the 9% of the final grade. This text includes several exercises, whose points are indicated at their beginning, possibly split among the set questions.
- Condition of assessment:
  This is an individual coursework, and it should be completed within the student's own time. Tutors may be consulted according to their meeting times and agreements.
- Specification:
  This coursework contains 8 exercises. Please provide clearly written solutions.
- Deliverables:
  Please submit your solutions on paper, bound securely.
- Marking Guidance:
  Single exercises in this text will be marked proportionally to the points they carry. The sum of all points obtained in this coursework will contribute to the points obtained at the end of all three courseworks.
- Feedback:
  It will be provided by example solutions, or by discussing solutions during tutorials.

Exercise 1 [points: 5, 5]
Consider the following formulae:

$$\begin{aligned} S_1 = & \ (\forall X)(\neg p(X, X)), \\ S_2 = & \ (\forall X)(\exists Y) p(X, Y), \\ S_3 = & \ (\forall X)(\forall Y)(\forall Z)(p(X, Y) \wedge p(Y, Z) \rightarrow p(X, Z)). \end{aligned}$$

1. Verify if the formulae above are simultaneously true in the interpretation I having domain the set of natural numbers $\mathbf{N}$ and mapping the symbol p as follows: $p^I(X, Y) = T$ iff $X > Y$.

2. Verify if the formulae above are simultaneously true in the interpretation I having domain the set of natural numbers $\mathbf{N}$ and mapping the symbol p as follows: $p^I(X, Y) = T$ iff $X \leq Y$.

Exercise 2 [points: 10, 10]
Let S be the formula $(\forall X)(p(X) \rightarrow p(f(X)))$.

1. Let I be the interpretation having the set of natural numbers $\mathbf{N}$ as domain and such that the predicate and function symbols p and f, respectively, are interpreted in I as follows: $p^I(X) = T$ iff X is an even natural number and $f^I(X) = X + 4$, for all $X \in \mathbf{N}$. Is I a model of S?

2. Let $D = \{a, b, c\}$, let I' be an interpretation having domain D and such that the predicate symbol p is interpreted as follows: $p^{I'}(a) = T$, $p^{I'}(b) = T$, $p^{I'}(c) = F$. Define $f^{I'}$, the interpretation of the symbol f in I' in such a way that the formula S is true in I'.

Exercise 3 [points: 5, 5, 5]
Given the formula $(A \leftrightarrow (B \vee C)) \leftrightarrow (B \vee \neg C)$ write its disjunctive normal form, its conjunctive normal form and its clausal normal form.

Exercise 4 [points: 10]
Prove formally the following statement: each formula is logically equivalent to some formula in prenex normal form.

Exercise 5 [points: 5]
Write the Skolem form of the following formula S:

$$(\forall Z)(\exists Y)[p(X, g(Y), Z) \vee \neg(\forall X) q(X)] \wedge \neg(\forall Z)(\exists X)(\neg r(f(X, Z), Z)).$$

Exercise 6 [points: 15]
Prove formally that if F and $\neg G$ are equivalent propositional formulae and F is satisfiable but not valid, then G is satisfiable but not valid.

Exercise 7 [points: 10]
Prove the formula $((\forall X)p(X) \vee (\forall X)q(X)) \rightarrow (\forall X)(p(X) \vee q(X))$ with the method of semantic tableaux.

Exercise 8 [points: 15]
Prove by resolution that the formula below is valid:

$$(\forall X)(\forall Z)[(\ (p(X) \rightarrow (\exists Y)q(Y)) \wedge p(X) \wedge ((\exists Y)q(Y) \rightarrow r(Z))\ ) \rightarrow r(Z)].$$