A PERSONAL PERSPECTIVE ON DEEP INFERENCE AND COMPUTER SCIENCE

ALESSIO GUGLIELMI

1. Introduction

Deep inference is a young area in proof theory, which is the discipline that studies mathematical proofs. It is a new methodology for designing proof formalisms that generalise those introduced by Gentzen, in the '30s, for the presentation of proof systems [Gen69]. The conceptual origins of deep inference are mainly in the proof-theory area of *linear logic*, initially developed by Girard, in the '80s [Gir87]. The future of deep inference tends towards proof complexity, combinatorics and the study of proofs via algebraic topology. One of the most important open problems that deep inference intends to solve is that of the *identity of proofs* (sometimes called *Hilbert's 24th problem* [Thi03]); this is related to the equally open problem of the *identity of algorithms* [BDG08].

Linear logic, among other ideas, supports the notion that logic has a *geometric* nature, and that a more perspicuous analysis of proofs is possible if we uncover their geometric shape, hidden behind their syntax. We can give technical meaning to this notion by looking for *linearity* in proofs. In the computing world, linearity can be interpreted as a way to deal with *quantity* or *resource*. The significance of linear logic for computer science has stimulated a remarkable amount of research, that continues to these days, and that ranges from the most theoretical investigations in categorical semantics to the implementation of languages and compilers, verification of software, and other practical applications.[1]

Linear logic expresses its forms of locality by relying on Gentzen's formalisms. However, these had been developed for classical mathematical logic, for which linearity is not a primitive, natural notion. While attempting to relate *process algebras* (which are foundational models of concurrent computation) to linear logic, I realised that Gentzen's formalisms were inherently inadequate to express the most primitive notion of composition in computer science: *sequential composition*. This is indeed linear, but of a different kind of linearity from that naturally supported by linear logic.

I thought then that the linear logic ideas were to be carried all the way through and that the formalisms themselves had to be 'linearised'. Technically, this turned out to be possible by dropping one of the assumptions that Gentzen implicitly used, namely that the (geometric) shape of proofs is directly related to the shape of formulae that they prove. In deep inference, we do not make this assumption, and we get proofs whose shape is much more liberally determined than in Gentzen's formalisms. As an immediate consequence, we were able to capture process-algebras sequential composition, but we soon realised that the new formalism was offering unprecedented opportunities for both a more satisfying general theory of proofs and for more applications in computer science.

This web page contains up-to-date information about deep inference:

`http://alessio.guglielmi.name/res/cos` .

Experts in proof theory might find the web page *Deep Inference in One Minute* helpful:

`http://alessio.guglielmi.name/res/cos/diom.html` .

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[1] A search for "linear logic" in Google Scholar returns 16,000 items.

2. Achievements

Together with my students and colleagues, I started, in 2001, a research programme whose objective was the definition of the simplest deep-inference formalism conceivable and the development of its proof theory. We called the formalism the *calculus of structures*, and, to a very large extent, its proof theory and its relations with computer science are now understood. This is a summary of the achievements so far, with special emphasis on computer science applications:

- *Analyticity and automated deduction.* Classical [Brü03a, Brü03b, Brü04, Brü06a, Brü06d, BG04, BT01, Brü10a], intuitionistic [Hor06, Pos09, Tiu06a], linear [Str03a, Str03b] and several modal logics [Brü10b, Brü09, Brü06c, BS09a, BS09b, GT07, GPT08, GPT09, GPT10, GPT11, Hei05, HS05, SS05, Sto04, Sto07, BGK10] are expressed in *analytic* systems. Contrary to deep inference, Gentzen's methodology has difficulties dealing with modal logics, to the point that for many of them no analytic proof systems outside of deep inference are known. In particular, modal logics B and K5, which don't enjoy analytic presentations in Gentzen's formalisms, are expressed by *simple* analytic systems [Brü06c]. Analyticity is the property that allows us to automatise the search for proofs, so, this property is fundamental for *automated deduction*. Proof search systems have been implemented for several logics [Kah04, Kah06a, Kah06b].
- *Non-commutativity and process algebras.* Mixed commutative/non-commutative linear logics BV [Gug07, Kah06c, Kah07] and NEL [GS01, GS02, GS11, Str03c, SG10] are expressed in analytic systems, and we proved that these logics cannot be expressed analytically in Gentzen's formalisms [Tiu06b]; these logics can directly capture *sequential composition* in process algebras [Bru02], which we now know to be impossible in Gentzen formalisms. They also capture the linear lambda calculus with explicit substitutions [Rov11, Rov]. Languages and implementations have been realised, based on these deep-inference notions [Kah, Kah05, Kah08a, Kah08b, Kah09, Rei07]. In computer science, process algebras are used to model *communication protocols*. The problem of proving the *security* of protocols has great theoretical and practical relevance, and many methods in security rely on proof-theory. The ability to express process algebras directly in logic is, then, a significant achievement.
- *Non-commutativity and causal quantum evolution.* System BV is able to describe quantum causal evolution [BGI$^+$10]. This physical phenomenon causes several problems to other logics which people considered for the task, like linear logic. The basic idea is to represent quantum states as propositions and interaction as axioms [BPS08]. There is a close relationship with categorical models of BV [BPS11].
- *Normalisation and language design.* We developed general and powerful normalisation techniques for deep-inference systems for the classical [Brü03a, Brü06a, GG08, Gun09, GGS10, Str07a], intuitionistic [BM08], linear [Str03b], BV [Gug07] and NEL [GS02, GS11, Kah06a, SG10] logics; we achieved new, unifying normalisation notions, in addition to the traditional cut elimination one [Brü06b, GGP10]. Normalisation is the key to the so-called *Curry-Howard correspondence*, which plays a fundamental role in the design of *functional languages*. The concept of *type*, which is one of the most successful programming concepts, relies on the Curry-Howard correspondence, and, ultimately, on good normalisation properties in the proof-theoretic foundation of language design.
- *Locality and semantics.* Most deep-inference deductive systems consist entirely of local inference rules [Brü06d, DG04, Gug07, Sto07, Str02, Tiu06a]; a *local* inference rule is one whose computational complexity is a constant. Locality is a difficult property to achieve, and it is almost never achievable with Gentzen's methods [Brü03b], because of the necessity of duplicating formulae of unbounded size (lack

of linearity). Thanks to locality, we could discover a new class of *proof nets* for classical and linear logic [LS05a, LS05b, LS06, Str05, Str07c, Str09a, Str09b, SL04, Str10]. Proof nets play a crucial role in understanding *semantics of proofs* (see [Gui06, Hug04, Joi05, Lam07, LS05a, LS06, McK05, McK06, Str07b] for deep-inference contributions). Semantics of *programming languages* and of proof systems are mostly developed in parallel and they share the same mathematical structures.

- *Proof and computational complexity.* Proof theory and *computational complexity* are intimately related (mainly via the 'NP vs coNP' problem and its derivatives). Some recent results show that deep inference allows for analytic formalisms that are exponentially more efficient than Gentzen ones [BG09, Jap08, Das11], so contributing to the research on complexity lower bounds. Normalisation in the calculus of structures exceeded expectations: The works [BGGP11, BGGP10, BGGP09, Jeř09] show that quasipolynomial analytic proofs can be obtained by normalisation, instead of exponential ones as expected.

3. Perspectives

We can consider the initial development of the calculus of structures basically completed. I am now turning my attention towards the design of an evolution of the calculus of structures that, while retaining all its good proof-theoretical and complexity properties, will be at the same time closer to semantics and implementations. The new formalism is currently dubbed *Formalism B* and I am defining it together with Tom Gundersen (Bath) and Michel Parigot (CNRS Paris). This formalism will be a fundamental step towards solving the problem of the identity of proofs.

Formalism B's proofs are, in essence, geometric objects, and we reason and manipulate them mainly via topological and graph-rewriting transformations. The difference between Formalism B systems and proof nets [Gir87] is that the latter are not a proof system (in the precise sense of proof complexity), and so, deduction cannot be efficiently and directly performed on them. On the contrary, Formalism B proofs can be subjected to standard automated deduction techniques, and so they offer a suitable basis for applications. Due to their geometric nature, they do not suffer from so-called *bureaucracy*, which is a phenomenon that plagues traditional formalisms in two ways: 1) it unnecessarily increases non-determinism in proof-search, and 2) it unnecessarily widens the gap between syntax and semantics of proofs.

Solving the problem of the identity of proofs requires the creation of a wide array of proof normalisation techniques for a bureaucracy-free formalism, so that, for any given notion of equality of proofs, appropriate representations and decision procedures can be designed. Formalism B is bureaucracy-free, and my plan is to contribute to and to stimulate its development, as I did with the calculus of structures. The calculus of structures showed the validity of the deep-inference methodology; I think that Formalism B will be the tool to fully exploit deep inference both in the theory and the practice of computation.

Formalism B will be the basic setting where to carry on several proof-theoretical, semantical and proof-complexity investigations. My intention is to promote it as the most flexible and universal formalism for the study of the structural properties of proofs of the widest variety of logics. As for the calculus of structures, applications will be explored in automated deduction, process algebras, language design and semantics of programming languages.

Moreover, I intend to stimulate the development of experimental tools that should help the research outlined above. Experience suggests that much of our research benefits from the study of complex examples of proofs, that are increasingly more difficult to generate by hand. We will continue the development of proof assistants based on *Maude* (I'm referring especially to the cited works of Ozan Kahramanoğulları).

The recent developments in proof complexity suggest that the study of the proof complexity of deep inference can be a highly rewarding subject. The nature of deep inference proofs allows us to access the deep combinatorial nature of boolean functions in a much more direct way than any other formalism. We should then expect to understand more of the structure of boolean functions. In particular, deep inference should help design classes of tautologies for the study of lower bounds. In fact, deep inference is more efficient, proof-complexity-wise, than any other formalism. This means that it is more demanding than all others in the quest of lower bounds, and this makes it attractive, because lower bounds for deep inference are universal.

As I have been doing so far, I intend to work in close contact with computer scientists involved in the design and semantics of programming languages and with mathematicians in combinatorics and algebra. For me, proof theory is, primarily, a tool to understand computation. More specifically, it is one of our best hopes to be, one day, able to master the complexity of computation in such a way that the design of computer systems will be a solid science.

4. Brief History and Funding

Paola Bruscoli (who is an active deep-inference researcher) and I have been employed at the Technische Universität Dresden from 1998 to 2005, and research in deep inference, under my supervision, was further financed by 3 PhD studentships (Brünnler, Straßburger and Kahramanoğulları), full time for 3 years, for a value of approximately €510,000 (DFG *Graduiertenkolleg 334* and *446*).

I am now employed at the University of Bath (after having been on leave at INRIA as research director of the project Démosthène, in 2009 and 2010).

I have been principal investigator for the £118,072 EPSRC project *Complexity and Non-determinism in Deep Inference* (total value, £147,591); I was principal investigator for two British Council 2-year travel schemes with Paris (*New Deductive Systems, Normalisation Methods and Semantics for Deep Inference*) and Dresden (*Analyticity and Proof Search for Modal Logics in Deep Inference*) for a value of £6,950 for my research group in Bath.

Deep inference enjoys the grant *Theory and Application of Deep Inference* (coordinated by my former student Lutz Straßburger) for €230,400, and a total project cost of €1,280,598, from *Agence Nationale de la Recherche* (France).

Deep inference enjoyed a grant of €750,448, of which €355,056 from the French *Agence Nationale de la Recherche* and the rest from INRIA, for a *Senior Chaire d'Excellence*, employing two research directors (myself, principal investigator, full time, and François Lamarche, for 25% of his time) and a researcher (Paola Bruscoli). The title of the project was *Démosthène—Identity and Geometric Essence of Proofs*.

Deep inference was one of the main themes of a further grant of €80,000 for 2009-2010 from INRIA, for an *Action de Recherche Collaborative*: a two-year project of the teams Calligramme/Démosthène and Parsifal (INRIA) and the Computer Science Department of the University of Bath. Title of the project: *REDO: Redesigning Logical Syntax*. I was principal investigator, with François Lamarche and Lutz Straßburger (coordinator).

Deep inference is one of the main themes of the project *Structural and Computational Proof Theory* (*Structural*), from 2010 to 2012 (*ANR-Programme Blanc International/FWF 2010*). This project runs at PPS (École Polytechnique), LIX (Paris VII), Innsbruck and Vienna. The project has a total value of €2,324,133. Michel Parigot is coordinator and principal investigator.

Several groups have small travel grants that keep the various laboratories connected.

Deep inference is currently, mainly being developed at the University of Bath, University of Bern, Australian National University in Canberra, University of Turin, INRIA–Nancy-Grand-Est in Nancy, INRIA–Saclay-Île-de-France and CNRS in Paris. Research groups elsewhere have been active on deep inference in the past. I have ongoing relations with all these laboratories. Since 2005, there have been at least two international workshops per year devoted to deep inference.

The following document is an updated bibliography on deep inference:

`http://alessio.guglielmi.name/res/TeX/BibTeX/di-biblio.pdf` .

References

[BDG08] Andreas Blass, Nachum Dershowitz, and Yuri Gurevich. When are two algorithms the same? Technical Report MSR-TR-2008-20, Microsoft Research, 2008. `ftp://ftp.research.microsoft.com/pub/tr/TR-2008-20.pdf`.

[BG04] Kai Brünnler and Alessio Guglielmi. A first order system with finite choice of premises. In Vincent Hendricks, Fabian Neuhaus, Stig Andur Pedersen, Uwe Scheffler, and Heinrich Wansing, editors, *First-Order Logic Revisited*, Logische Philosophie, pages 59–74. Logos Verlag, 2004. `http://www.iam.unibe.ch/~kai/Papers/FinitaryFOL.pdf`.

[BG09] Paola Bruscoli and Alessio Guglielmi. On the proof complexity of deep inference. *ACM Transactions on Computational Logic*, 10(2):1–34, 2009. Article 14. `http://cs.bath.ac.uk/ag/p/PrComplDI.pdf`.

[BGGP09] Paola Bruscoli, Alessio Guglielmi, Tom Gundersen, and Michel Parigot. Quasipolynomial normalisation in deep inference via atomic flows and threshold formulae. `http://cs.bath.ac.uk/ag/p/QuasiPolNormDI.pdf`, 2009.

[BGGP10] Paola Bruscoli, Alessio Guglielmi, Tom Gundersen, and Michel Parigot. A quasipolynomial cut-elimination procedure in deep inference via atomic flows and threshold formulae. In Edmund M. Clarke and Andrei Voronkov, editors, *LPAR-16*, volume 6355 of *Lecture Notes in Computer Science*, pages 136–153. Springer-Verlag, 2010. `http://cs.bath.ac.uk/ag/p/QPNDI.pdf`.

[BGGP11] Paola Bruscoli, Alessio Guglielmi, Tom Gundersen, and Michel Parigot. A quasipolynomial normalisation procedure in deep inference. Submitted. `http://cs.bath.ac.uk/ag/p/QPNPDI.pdf`, 2011.

[BGI$^+$10] Richard F. Blute, Alessio Guglielmi, Ivan T. Ivanov, Prakash Panangaden, and Lutz Straßburger. A logical basis for quantum evolution and entanglement. Submitted, 2010.

[BGK10] Kai Brünnler, Remo Goetschi, and Roman Kuznets. A syntactic realization theorem for justification logics. In Lev Beklemishev, Valentin Goranko, and Valentin Shehtman, editors, *Advances in Modal Logic*, volume 8, pages 39–58. College Publications, 2010. `http://www.iam.unibe.ch/~kai/Papers/2010srtjl.pdf`.

[BM08] Kai Brünnler and Richard McKinley. An algorithmic interpretation of a deep inference system. In Iliano Cervesato, Helmut Veith, and Andrei Voronkov, editors, *LPAR 2008*, volume 5330 of *Lecture Notes in Computer Science*, pages 482—496. Springer-Verlag, 2008. `http://www.iam.unibe.ch/~kai/Papers/2008aidis.pdf`.

[BPS08] Rick Blute, Prakash Panangaden, and Lutz Straßburger. The logic BV and quantum causality. In *Trends in Logic VI: Logic and the Foundations of Physics: Space, Time and Quanta*, 2008. `http://www.lix.polytechnique.fr/~lutz/papers/BVlocative.pdf`.

[BPS11] Richard Blute, Prakash Panangaden, and Sergey Slavnov. Deep inference and probabilistic coherence spaces. *Applied Categorical Structures*, 2011. In press. `http://www.cs.mcgill.ca/~prakash/Pubs/seqcats.pdf`.

[Bru02] Paola Bruscoli. A purely logical account of sequentiality in proof search. In Peter J. Stuckey, editor, *Logic Programming, 18th International Conference*, volume 2401 of *Lecture Notes in Computer Science*, pages 302–316. Springer-Verlag, 2002. `http://cs.bath.ac.uk/pb/bvl/bvl.pdf`.

[Brü03a] Kai Brünnler. Atomic cut elimination for classical logic. In M. Baaz and J. A. Makowsky, editors, *CSL 2003*, volume 2803 of *Lecture Notes in Computer Science*, pages 86–97. Springer-Verlag, 2003. `http://www.iam.unibe.ch/~kai/Papers/ace.pdf`.

[Brü03b] Kai Brünnler. Two restrictions on contraction. *Logic Journal of the IGPL*, 11(5):525–529, 2003. `http://www.iam.unibe.ch/~kai/Papers/RestContr.pdf`.

[Brü04] Kai Brünnler. *Deep Inference and Symmetry in Classical Proofs*. Logos Verlag, Berlin, 2004. `http://www.iam.unibe.ch/~kai/Papers/phd.pdf`.

[Brü06a] Kai Brünnler. Cut elimination inside a deep inference system for classical predicate logic. *Studia Logica*, 82(1):51–71, 2006. `http://www.iam.unibe.ch/~kai/Papers/q.pdf`.

[Brü06b] Kai Brünnler. Deep inference and its normal form of derivations. In Arnold Beckmann, Ulrich Berger, Benedikt Löwe, and John V. Tucker, editors, *Logical Approaches to Computational Barriers—2nd Conference on Computability in Europe*, volume 3988 of *Lecture Notes in Computer Science*, pages 65–74. Springer-Verlag, 2006. `http://www.iam.unibe.ch/~kai/Papers/n.pdf`.

[Brü06c] Kai Brünnler. Deep sequent systems for modal logic. In Guido Governatori, Ian Hodkinson, and Yde Venema, editors, *Advances in Modal Logic*, volume 6, pages 107–119. College Publications, 2006. `http://www.aiml.net/volumes/volume6/Bruennler.ps`.

[Brü06d] Kai Brünnler. Locality for classical logic. *Notre Dame Journal of Formal Logic*, 47(4):557–580, 2006. http://www.iam.unibe.ch/~kai/Papers/LocalityClassical.pdf.

[Brü09] Kai Brünnler. Deep sequent systems for modal logic. *Archive for Mathematical Logic*, 48(6):551–577, 2009. http://www.iam.unibe.ch/~kai/Papers/2009dssml.pdf.

[Brü10a] Kai Brünnler. How to universally close the existential rule. In Christian G. Fermüller and Andrei Voronkov, editors, *LPAR-17*, volume 6397 of *Lecture Notes in Computer Science*, pages 172–186. Springer-Verlag, 2010. http://www.iam.unibe.ch/~kai/Papers/2010hucer.pdf.

[Brü10b] Kai Brünnler. Nested sequents. Habilitation Thesis. http://arxiv.org/pdf/1004.1845, 2010.

[BS09a] Kai Brünnler and Lutz Straßburger. Modular sequent systems for modal logic. In *Tableaux 2009*, volume 5607 of *Lecture Notes in Artificial Intelligence*, pages 152–166. Springer-Verlag, 2009. http://www.iam.unibe.ch/~kai/Papers/2009mssml.pdf.

[BS09b] Kai Brünnler and Thomas Studer. Syntactic cut-elimination for common knowledge. *Annals of Pure and Applied Logic*, 160:82–95, 2009. http://www.iam.unibe.ch/~kai/Papers/infck.pdf.

[BT01] Kai Brünnler and Alwen Fernanto Tiu. A local system for classical logic. In R. Nieuwenhuis and Andrei Voronkov, editors, *LPAR 2001*, volume 2250 of *Lecture Notes in Computer Science*, pages 347–361. Springer-Verlag, 2001. http://www.iam.unibe.ch/~kai/Papers/lcl-lpar.pdf.

[Das11] Anupam Das. On the proof complexity of cut-free bounded deep inference. In Kai Brünnler and George Metcalfe, editors, *Tableaux 2011*, Lecture Notes in Artificial Intelligence. Springer-Verlag, 2011. In press.

[DG04] Pietro Di Gianantonio. Structures for multiplicative cyclic linear logic: Deepness vs cyclicity. In J. Marcinkowski and A. Tarlecki, editors, *CSL 2004*, volume 3210 of *Lecture Notes in Computer Science*, pages 130–144. Springer-Verlag, 2004. http://www.dimi.uniud.it/~pietro/papers/Soft-copy-ps/scll.ps.gz.

[Gen69] Gerhard Gentzen. Investigations into logical deduction. In M. E. Szabo, editor, *The Collected Papers of Gerhard Gentzen*, pages 68–131. North-Holland, Amsterdam, 1969.

[GG08] Alessio Guglielmi and Tom Gundersen. Normalisation control in deep inference via atomic flows. *Logical Methods in Computer Science*, 4(1:9):1–36, 2008. http://www.lmcs-online.org/ojs/viewarticle.php?id=341.

[GGP10] Alessio Guglielmi, Tom Gundersen, and Michel Parigot. A proof calculus which reduces syntactic bureaucracy. In Christopher Lynch, editor, *21st International Conference on Rewriting Techniques and Applications*, volume 6 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 135–150. Schloss Dagstuhl–Leibniz-Zentrum für Informatik, 2010. http://drops.dagstuhl.de/opus/volltexte/2010/2649.

[GGS10] Alessio Guglielmi, Tom Gundersen, and Lutz Straßburger. Breaking paths in atomic flows for classical logic. In Jean-Pierre Jouannaud, editor, *25th Annual IEEE Symposium on Logic in Computer Science*, pages 284–293. IEEE, 2010. http://www.lix.polytechnique.fr/~lutz/papers/AFII.pdf.

[Gir87] Jean-Yves Girard. Linear logic. *Theoretical Computer Science*, 50:1–102, 1987.

[GPT08] Rajeev Goré, Linda Postniece, and Alwen Tiu. Cut-elimination and proof-search for bi-intuitionistic logic using nested sequents. In Carlos Areces and Robert Goldblatt, editors, *Advances in Modal Logic*, volume 7, pages 43–66. College Publications, 2008. http://www.aiml.net/volumes/volume7/Gore-Postniece-Tiu.pdf.

[GPT09] Rajeev Goré, Linda Postniece, and Alwen Tiu. Taming displayed tense logics using nested sequents with deep inference. In *Tableaux 2009*, volume 5607 of *Lecture Notes in Artificial Intelligence*, pages 189–204. Springer-Verlag, 2009. http://users.cecs.anu.edu.au/~tiu/papers/kt.pdf.

[GPT10] Rajeev Goré, Linda Postniece, and Alwen Tiu. Cut-elimination and proof search for bi-intuitionistic tense logic. In Lev Beklemishev, Valentin Goranko, and Valentin Shehtman, editors, *Advances in Modal Logic*, volume 8, pages 156–177. College Publications, 2010. http://www.aiml.net/volumes/volume8/Gore-Postniece-Tiu.pdf.

[GPT11] Rajeev Goré, Linda Postniece, and Alwen Tiu. On the correspondence between display postulates and deep inference in nested sequent calculi for tense logics. *Mathematical Structures in Computer Science*, 2011. In press. http://arxiv.org/pdf/1103.5286.

[GS01] Alessio Guglielmi and Lutz Straßburger. Non-commutativity and MELL in the calculus of structures. In L. Fribourg, editor, *CSL 2001*, volume 2142 of *Lecture Notes in Computer Science*, pages 54–68. Springer-Verlag, 2001. http://cs.bath.ac.uk/ag/p/NoncMELLCoS.pdf.

[GS02] Alessio Guglielmi and Lutz Straßburger. A non-commutative extension of MELL. In Matthias Baaz and Andrei Voronkov, editors, *LPAR 2002*, volume 2514 of *Lecture Notes in Computer Science*, pages 231–246. Springer-Verlag, 2002. http://www.lix.polytechnique.fr/~lutz/papers/NEL.pdf.

[GS11] Alessio Guglielmi and Lutz Straßburger. A system of interaction and structure V: The exponentials and splitting. *Mathematical Structures in Computer Science*, 21(3):563–584, 2011. http://www.lix.polytechnique.fr/~lutz/papers/NEL-splitting.pdf.

[GT07] Rajeev Goré and Alwen Tiu. Classical modal display logic in the calculus of structures and minimal cut-free deep inference calculi for S5. *Journal of Logic and Computation*, 17(4):767–794, 2007. http://users.cecs.anu.edu.au/~tiu/papers/cmdl.pdf.

[Gug07] Alessio Guglielmi. A system of interaction and structure. *ACM Transactions on Computational Logic*, 8(1):1–64, 2007. `http://cs.bath.ac.uk/ag/p/SystIntStr.pdf`.

[Gui06] Yves Guiraud. The three dimensions of proofs. *Annals of Pure and Applied Logic*, 141(1-2):266–295, 2006. `http://www.loria.fr/~guiraudy/recherche/cos1.pdf`.

[Gun09] Tom Gundersen. *A General View of Normalisation Through Atomic Flows*. PhD thesis, University of Bath, 2009. `http://tel.archives-ouvertes.fr/docs/00/50/92/41/PDF/thesis.pdf`.

[Hei05] Robert Hein. Geometric theories and modal logic in the calculus of structures. Master's thesis, Technische Universität Dresden, 2005. `http://bitschnitzer.de/robert_thesis.ps.gz`.

[Hor06] Benjamin Robert Horsfall. The logic of bunched implications: A memoir. Master's thesis, University of Melbourne, 2006. `http://www.cs.mu.oz.au/~horsfall/horsfall-mcs.pdf`.

[HS05] Robert Hein and Charles Stewart. Purity through unravelling. In Paola Bruscoli, François Lamarche, and Charles Stewart, editors, *Structures and Deduction*, pages 126–143. Technische Universität Dresden, 2005. ICALP Workshop. ISSN 1430-211X. `http://cs.bath.ac.uk/pb/SD05/SD05-Proc.pdf`.

[Hug04] Dominic J.D. Hughes. Deep inference proof theory equals categorical proof theory minus coherence. `http://boole.stanford.edu/~dominic/papers/di/di.pdf`, 2004.

[Jap08] Giorgi Japaridze. Cirquent calculus deepened. *Journal of Logic and Computation*, 18(6):983–1028, 2008. `http://arxiv.org/pdf/0709.1308`.

[Jeř09] Emil Jeřábek. Proof complexity of the cut-free calculus of structures. *Journal of Logic and Computation*, 19(2):323–339, 2009. `http://www.math.cas.cz/~jerabek/papers/cos.pdf`.

[Joi05] Jean-Baptiste Joinet. Completeness of MLL proof-nets w.r.t. weak distributivity. In Paola Bruscoli, François Lamarche, and Charles Stewart, editors, *Structures and Deduction*, pages 81–94. Technische Universität Dresden, 2005. ICALP Workshop. ISSN 1430-211X. Invited talk at WoLLIC '03 under the title 'Calculus of Structures and Proof-Nets'. `http://cs.bath.ac.uk/pb/SD05/SD05-Proc.pdf`.

[Kah] Ozan Kahramanoğulları. Labeled event structure semantics of linear logic planning. Presented at the 1st World Congress on Universal Logic. `http://www.doc.ic.ac.uk/~ozank/Papers/unilog.pdf`.

[Kah04] Ozan Kahramanoğulları. System BV without the equalities for unit. In C. Aykanat, T. Dayar, and I. Körpeoğlu, editors, *19th International Symposium on Computer and Information Sciences, ISCIS 2004*, volume 3280 of *Lecture Notes in Computer Science*, pages 986–995. Springer-Verlag, 2004. `http://www.doc.ic.ac.uk/~ozank/Papers/bvn.pdf`.

[Kah05] Ozan Kahramanoğulları. Towards planning as concurrency. In *The IASTED International Conference on Artificial Intelligence and Applications, AIA 2005*, 2005. `http://www.doc.ic.ac.uk/~ozank/Papers/aia05.pdf`.

[Kah06a] Ozan Kahramanoğulları. *Nondeterminism and Language Design in Deep Inference*. PhD thesis, Technische Universität Dresden, 2006. `http://www.doc.ic.ac.uk/~ozank/Papers/ozansthesis.pdf`.

[Kah06b] Ozan Kahramanoğulları. Reducing nondeterminism in the calculus of structures. In Miki Hermann and Andrei Voronkov, editors, *LPAR 2006*, volume 4246 of *Lecture Notes in Computer Science*, pages 272–286. Springer-Verlag, 2006. `http://dx.doi.org/10.1007/11916277_19`.

[Kah06c] Ozan Kahramanoğulları. System BV is NP-complete. In Ruy de Queiroz, Angus Macintyre, and Guilherme Bittencourt, editors, *12th Workshop on Logic, Language, Information and Computation (WoLLIC 2005)*, volume 143 of *Electronic Notes in Theoretical Computer Science*, pages 87–99. Elsevier, 2006. `http://www.doc.ic.ac.uk/~ozank/Papers/BVnpc.pdf`.

[Kah07] Ozan Kahramanoğulları. System BV is NP-complete. *Annals of Pure and Applied Logic*, 152(1–3):107–121, 2007. `http://dx.doi.org/10.1016/j.apal.2007.11.005`.

[Kah08a] Ozan Kahramanoğulları. Maude as a platform for designing and implementing deep inference systems. In J. Visser and V. Winter, editors, *Eighth International Workshop on Rule Based Programming (RULE 2007)*, volume 219 of *Electronic Notes in Theoretical Computer Science*, pages 35–50. Elsevier, 2008. `http://www.doc.ic.ac.uk/~ozank/Papers/rule07.pdf`.

[Kah08b] Ozan Kahramanoğulları. On linear logic planning and concurrency. In Carlos Martín-Vide, Friedrich Otto, and Henning Fernau, editors, *Language and Automata Theory and Applications*, volume 5196 of *Lecture Notes in Computer Science*, pages 250–262. Springer-Verlag, 2008. `http://www.doc.ic.ac.uk/~ozank/Papers/lata.pdf`.

[Kah09] Ozan Kahramanoğulları. On linear logic planning and concurrency. *Information and Computation*, 207(11):1229–1258, 2009. `http://www.doc.ic.ac.uk/~ozank/Papers/on_linear.pdf`.

[Lam07] François Lamarche. Exploring the gap between linear and classical logic. *Theory and Applications of Categories*, 18(17):473–535, 2007. `http://www.loria.fr/~lamarche/papers/Gap.pdf`.

[LS05a] François Lamarche and Lutz Straßburger. Constructing free boolean categories. In Prakash Panangaden, editor, *20th Annual IEEE Symposium on Logic in Computer Science*, pages 209–218. IEEE, 2005. `http://www.lix.polytechnique.fr/~lutz/papers/FreeBool-long.pdf`.

[LS05b] François Lamarche and Lutz Straßburger. Naming proofs in classical propositional logic. In Pawe Urzyczyn, editor, *Typed Lambda Calculi and Applications*, volume 3461 of *Lecture Notes in Computer Science*, pages 246–261. Springer-Verlag, 2005. `http://www.lix.polytechnique.fr/~lutz/papers/namingproofsCL.pdf`.

[LS06] François Lamarche and Lutz Straßburger. From proof nets to the free *-autonomous category. *Logical Methods in Computer Science*, 2(4):3:1–44, 2006. `http://arxiv.org/pdf/cs.LO/0605054`.

[McK05] Richard McKinley. Classical categories and deep inference. In Paola Bruscoli, François Lamarche, and Charles Stewart, editors, *Structures and Deduction*, pages 19–33. Technische Universität Dresden, 2005. ICALP Workshop. ISSN 1430-211X. `http://www.cs.bath.ac.uk/~csprim/csl05.pdf`.

[McK06] Richard McKinley. *Categorical Models of First Order Classical Proofs*. PhD thesis, University of Bath, 2006. `http://www.cs.bath.ac.uk/~csprim/thesis.pdf`.

[Pos09] Linda Postniece. Deep inference in bi-intuitionistic logic. In Hiroakira Ono, Makoto Kanazawa, and Ruy de Queiroz, editors, *16th Workshop on Logic, Language, Information and Computation (WoLLIC 2009)*, volume 5514 of *Lecture Notes in Computer Science*, pages 320–334. Springer-Verlag, 2009. `http://users.cecs.anu.edu.au/~linda/postniece-dbi.pdf`.

[Rei07] Antoine Reilles. Canonical abstract syntax trees. In *6th International Workshop on Rewriting Logic and Its Applications*, volume 176 of *Electronic Notes in Theoretical Computer Science*, pages 165–179. Elsevier, 2007. `http://arxiv.org/pdf/cs.PL/0601019`.

[Rov] Luca Roversi. Linear lambda calculus with explicit substitutions as proof-search in deep inference. `http://arxiv.org/pdf/1011.3668`.

[Rov11] Luca Roversi. Linear lambda calculus and deep inference. In Luke Ong, editor, *Typed Lambda Calculi and Applications*, volume 6690 of *Lecture Notes in Computer Science*, pages 184–197. Springer-Verlag, 2011. `http://www.di.unito.it/~rover/RESEARCH/PUBLICATIONS/2011-TLCA/Roversi2011TLCA.pdf`.

[SG10] Lutz Straßburger and Alessio Guglielmi. A system of interaction and structure IV: The exponentials and decomposition. *ACM Transactions on Computational Logic*, 2010. In press. `http://arxiv.org/pdf/0903.5259`.

[SL04] Lutz Straßburger and François Lamarche. On proof nets for multiplicative linear logic with units. In J. Marcinkowski and A. Tarlecki, editors, *CSL 2004*, volume 3210 of *Lecture Notes in Computer Science*, pages 145–159. Springer-Verlag, 2004. `http://www.lix.polytechnique.fr/~lutz/papers/multPN.pdf`.

[SS05] Charles Stewart and Phiniki Stouppa. A systematic proof theory for several modal logics. In Renate Schmidt, Ian Pratt-Hartmann, Mark Reynolds, and Heinrich Wansing, editors, *Advances in Modal Logic*, volume 5, pages 309–333. King's College Publications, 2005. `http://www.aiml.net/volumes/volume5/Stewart.ps`.

[Sto04] Phiniki Stouppa. The design of modal proof theories: The case of S5. Master's thesis, Technische Universität Dresden, 2004. `http://www.wv.inf.tu-dresden.de/Publications/Diploma/diplom_stouppa.pdf`.

[Sto07] Phiniki Stouppa. A deep inference system for the modal logic S5. *Studia Logica*, 85(2):199–214, 2007. `http://www.iam.unibe.ch/til/publications/pubitems/pdfs/sto07.pdf`.

[Str02] Lutz Straßburger. A local system for linear logic. In Matthias Baaz and Andrei Voronkov, editors, *LPAR 2002*, volume 2514 of *Lecture Notes in Computer Science*, pages 388–402. Springer-Verlag, 2002. `http://www.lix.polytechnique.fr/~lutz/papers/lls-lpar.pdf`.

[Str03a] Lutz Straßburger. *Linear Logic and Noncommutativity in the Calculus of Structures*. PhD thesis, Technische Universität Dresden, 2003. `http://www.lix.polytechnique.fr/~lutz/papers/dissvonlutz.pdf`.

[Str03b] Lutz Straßburger. MELL in the calculus of structures. *Theoretical Computer Science*, 309:213–285, 2003. `http://www.lix.polytechnique.fr/~lutz/papers/els.pdf`.

[Str03c] Lutz Straßburger. System NEL is undecidable. In Ruy De Queiroz, Elaine Pimentel, and Lucília Figueiredo, editors, *10th Workshop on Logic, Language, Information and Computation (WoLLIC)*, volume 84 of *Electronic Notes in Theoretical Computer Science*. Elsevier, 2003. `http://www.lix.polytechnique.fr/~lutz/papers/NELundec_wollic03.pdf`.

[Str05] Lutz Straßburger. From deep inference to proof nets. In Paola Bruscoli, François Lamarche, and Charles Stewart, editors, *Structures and Deduction*, pages 2–18. Technische Universität Dresden, 2005. ICALP Workshop. ISSN 1430-211X. `http://www.lix.polytechnique.fr/~lutz/papers/deepnet-SD05.pdf`.

[Str07a] Lutz Straßburger. A characterisation of medial as rewriting rule. In Franz Baader, editor, *Term Rewriting and Applications—18th International Conference, RTA*, volume 4533 of *Lecture Notes in Computer Science*, pages 344–358. Springer-Verlag, 2007. `http://www.lix.polytechnique.fr/~lutz/papers/CharMedial.pdf`.

[Str07b] Lutz Straßburger. On the axiomatisation of boolean categories with and without medial. *Theory and Applications of Categories*, 18(18):536–601, 2007. `http://www.lix.polytechnique.fr/~lutz/papers/medial.pdf`.

[Str07c] Lutz Straßburger. What is a logic, and what is a proof? In Jean-Yves Beziau, editor, *Logica Universalis—Towards a General Theory of Logic*, pages 135–152. Birkhäuser, 2007. `http://www.lix.polytechnique.fr/~lutz/papers/WhatLogicProof.pdf`.

[Str09a] Lutz Straßburger. From deep inference to proof nets via cut elimination. *Journal of Logic and Computation*, 2009. In press. `http://www.lix.polytechnique.fr/~lutz/papers/deepnet.pdf`.

[Str09b] Lutz Straßburger. Some observations on the proof theory of second order propositional multiplicative linear logic. In Pierre-Louis Curien, editor, *Typed Lambda Calculi and Applications*, volume 5608 of *Lecture Notes in Computer Science*, pages 309–324. Springer-Verlag, 2009. `http://www.lix.polytechnique.fr/~lutz/papers/ObsPT-MLL2-finalforTLCA09.pdf`.

[Str10] Lutz Straßburger. What is the problem with proof nets for classical logic? In Fernando Ferreira, Benedikt Löwe, Elvira Mayordomo, and Luís Mendes Gomes, editors, *Programs, Proofs, Processes—6th Conference on Computability in Europe*, volume 6158 of *Lecture Notes in Computer Science*, pages 406–416. Springer-Verlag, 2010. `http://www.lix.polytechnique.fr/~lutz/papers/CiE10.pdf`.

[Thi03] Rüdiger Thiele. Hilbert's twenty-fourth problem. *American Mathematical Monthly*, 110:1–24, 2003.

[Tiu06a] Alwen Tiu. A local system for intuitionistic logic. In Miki Hermann and Andrei Voronkov, editors, *LPAR 2006*, volume 4246 of *Lecture Notes in Computer Science*, pages 242–256. Springer-Verlag, 2006. `http://users.cecs.anu.edu.au/~tiu/localint.pdf`.

[Tiu06b] Alwen Tiu. A system of interaction and structure II: The need for deep inference. *Logical Methods in Computer Science*, 2(2:4):1–24, 2006. `http://arxiv.org/pdf/cs.LO/0512036`.

University of Bath `http://alessio.guglielmi.name`