Consider the following linear inference from [Das13]:

\[
\begin{align*}
(a \lor (b \land b')) \land [c \land (d' \lor d')] \land [(e \land e') \lor f] \\
(a \land [c \lor e]) \lor (c' \land e') \lor (b' \land d') \lor ([b \lor d] \land f)
\end{align*}
\]

Recall that this is not derivable in \{s, m\}. Indeed there could be no final step (in CoS-style), since any step would break soundness. It remains underviable in the presence of units, since they can only help when there is some triviality around (see [Das13]), which is not the case by inspection of the semantics.

It turns out we may derive it using a single contraction loop, on either \(a\) or \(f\).

The derivation for \(a\) is below, the one for \(f\) being symmetric:

\[
\begin{align*}
& \frac{\text{act} \quad \frac{a}{a \land a} \quad \frac{a \lor (b \land b')} {a \lor b} \land [a \lor b']} {m} \\
& \frac{\frac{a \lor b} {a \lor b} \land [a \lor b']} {2s} \\
& \frac{\frac{(a \land c) \lor [b \lor d] \land f} {a \land c} \lor ([b \lor d] \land f)} {s} \\
& \frac{\frac{(a \land c) \lor [b \lor d] \land f} {a \land c} \lor ([b \lor d] \land f)} {2s} \\
& \frac{\frac{(a \land c) \lor [b \lor d] \land f} {a \land c} \lor ([b \lor d] \land f)} {2s} \\
& \frac{\frac{(a \land c) \lor [b \lor d] \land f} {a \land c} \lor ([b \lor d] \land f)} {2s} \\
& \frac{\frac{(a \land c) \lor [b \lor d] \land f} {a \land c} \lor ([b \lor d] \land f)} {2s} \\
& \frac{\frac{(a \land c) \lor [b \lor d] \land f} {a \land c} \lor ([b \lor d] \land f)} {2s} \\
\end{align*}
\]

Here we have used different colours to distinguish the two \(a\)-paths forming the loop. This contraction loop cannot be eliminated, since \(1\) is underviable in \{s, m\}. Indeed, even if we added \(1\) to \textsc{SKS}, we could find another similar situation with a more complicated inference, and so on for any (non-deterministic) polynomial-time decidable system extending \textsc{SKS} by linear rules unless \textsc{coNP} = \textsc{NP}, due to [DS16].

Notice from the above that \(1\) is not actually minimal, due to the available application of medial to the middle conjunct of the premiss. The resulting linear inference is indeed minimal.

\section*{References}


\textit{Date}: June 25, 2017.

\footnote{For this, it is helpful to appeal to the pigeonhole intuition from [Das13].}