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% CM30071 COURSEWORK
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%
% Topic 4
%
% We want to show that first order logic is undecidable by using the known fact
% that the Post Correspondence Problem (PCP) is undecidable.
%
% Please see in the next page a definition and example of PCP, and consider a
% given the fact that the problem is undecidable.
%
% Below, you find a Prolog program which solves the PCP and which is also a
% theory in first order logic. If first order logic was decidable, PCP would be
% decidable, too, which is absurd.


% Encoding of PCP: 1 - the two sets of words (this corresponds to the example
% given in the following page):

string_W(      s(0),Y,        g(Y)).
string_W(s(s(0))   ,Y,g(f(g(g(g(Y)))))).
string_W(s(s(s(0))),Y,      g(f(Y))).

string_X(      s(0),Y,g(g(g(Y)))).
string_X(   s(s(0)),Y,  g(f(Y))).
string_X(s(s(s(0))),Y,    f(Y)).


% Encoding of PCP: 2 - definition of the problem:

pcp_init(W,X) :- string_W(I,0,W),
                 string_X(I,0,X).

pcp_aux(W,X) :- pcp_init(W,X).
pcp_aux(W,X) :- pcp_aux(W1,X1),
                string_W(I,W1,W),
                string_X(I,X1,X).

pcp(S) :- pcp_aux(S,S).


% Encoding of PCP: 3 - the goal:

:- pcp(S).


% This defines the PCP in pure Prolog, which corresponds also to a first order
% predicate logic theory.
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8.5 UNDECIDABILITY OF POST'S CORRESPONDENCE PROBLEM

Undecidable problems arise in a variety of areas. In the next three sections we explore some of the more interesting problems in language theory and develop techniques for proving particular problems undecidable. We begin with Post's Correspondence Problem, it being a valuable tool in establishing other problems to be undecidable.

An instance of *Post's Correspondence Problem* (*PCP*) consists of two lists, $A = w_1, \ldots, w_k$ and $B = x_1, \ldots, x_k$, of strings over some alphabet Σ. This instance of PCP *has a solution* if there is any sequence of integers $i_1, i_2, \ldots, i_m$, with $m \geq 1$, such that

$$w_{i_1}, w_{i_2}, \ldots, w_{i_m} = x_{i_1}, x_{i_2}, \ldots, x_{i_m}.$$

The sequence $i_1, \ldots, i_m$ is a solution to this instance of PCP.

Example 8.6 Let $\Sigma = \{0, 1\}$. Let A and B be lists of three strings each, as defined in Fig. 8.15. In this case PCP has a solution. Let $m = 4$, $i_1 = 2$, $i_2 = 1$, $i_3 = 1$, and $i_4 = 3$. Then

$$w_2 w_1 w_1 w_3 = x_2 x_1 x_1 x_3 = 101111110.$$

	List A	List B
i	w_i	x_i
1	1	111
2	10111	10
3	10	0

Fig. 8.15 An instance of PCP.