University of Bath

DEPARTMENT OF COMPUTER SCIENCE EXAMINATION

CM30071: LOGIC AND ITS APPLICATIONS

CLASSROOM TEST—NOT AN EXAM!!!
Please ignore the instructions below, these are only valid for the real examination.
You should be able to solve these two exercises in 80 minutes.

Full marks will be given for correct answers to THREE questions. If you opt to answer more than the specified number of questions, you should clearly identify which of your answers you wish to have marked. In cases where you have failed to identify the correct number of answers the marker is only obliged to consider the answers in the order they appear up to the number of answers required.

1. (a) Show a sequent calculus system for propositional classical logic; use any notation you like, provided the system is complete. [4]

   (b) Consider, for every $h \geq 1$, the formula

$$
\begin{array}{rll}
F_h = & (& (a_1 \vee b_1) \wedge \\
& & (a_1 \rightarrow (a_2 \vee b_2)) \wedge (b_1 \rightarrow (a_2 \vee b_2)) \wedge \\
& & \cdots \wedge \\
& & (a_{h-1} \rightarrow (a_h \vee b_h)) \wedge (b_{h-1} \rightarrow (a_h \vee b_h)) \\
&) & \rightarrow \\
& & (a_h \vee b_h) \quad .
\end{array}
$$

   For example, F_1 and F_2 are:

$$
\begin{array}{l}
F_1 = (a_1 \vee b_1) \rightarrow (a_1 \vee b_1) \quad , \\
F_2 = ((a_1 \vee b_1) \wedge (a_1 \rightarrow (a_2 \vee b_2)) \wedge (b_1 \rightarrow (a_2 \vee b_2))) \rightarrow (a_2 \vee b_2) \quad .
\end{array}
$$

   Prove that every F_h is a tautology by using the sequent calculus system that you provided before. Note: to get full marks, it is necessary to provide a concise solution; please feel free to define pieces of derivations you might need, give them a name and then use the names instead of replicating many derivations. [16]

2. (a) Show a natural deduction system for propositional classical logic; use any notation you like, provided the system is complete. [4]

   (b) Prove, in the system you gave, that

$$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)) \quad .$$

   [6]

   (c) Prove, in the system you gave, that

$$((p \wedge q) \wedge \neg(p \vee q)) \rightarrow r.$$

   [10]