

**CM20019—COMPUTATION III:  
FORMAL LOGIC AND SEMANTICS  
EXERCISE SHEET 6, 6.11.2007**

ALESSIO GUGLIELMI

**Problem 1.** Express the following statements into formulae of some first order language.

- (1) Not all toothless animals which have feathers can fly.
- (2) If some human being is in prison than all human beings are in prison.
- (3) It is not true that every tall person who is not crazy likes some other person who is either not tall or crazy.
- (4) Gorillas are hairier than billiard balls.
- (5) Everyone is either loved or hated by some other person.
- (6) There is a town in Spain in which there is a barber who shaves everyone in the town who does not shave themselves.
- (7) Everyone is happy except for Jane.
- (8) There is only one thing with wheels in the shed and that is a bicycle with a flat tire.
- (9) For every  $X$  and  $Y$  there is a  $Z$  so that  $X \leq Z$  and  $Y \leq Z$ , and if  $W$  is such that  $X \leq W$  and  $Y \leq W$  then  $Z \leq W$ .

**Problem 2.** For each of the following four schemes of formulae, find formulae  $F$  and  $G$  and an interpretation  $I$  such that, when instantiated with the formulae you found, the scheme yields a formula not valid in  $I$ :

- (1)  $(\exists X)F \rightarrow (\forall X)F$  ;
- (2)  $(\forall X)(\exists Y)F \rightarrow (\exists Y)(\forall X)F$  ;
- (3)  $((\exists X)F \wedge (\exists X)G) \rightarrow (\exists X)(F \wedge G)$  ;
- (4)  $(\forall X)(F \vee G) \rightarrow ((\forall X)F \vee (\forall X)G)$  .

\* \* \*

The web page for the course is at [1]. You can find other exercises in Dan Richardson's notes, available from the web page.

References

1. Alessio Guglielmi, *CM20019—Computation III: Formal logic and semantics*, <http://cs.bath.ac.uk/ag/CM20019>, 2007.