

**CM20019—COMPUTATION III:  
FORMAL LOGIC AND SEMANTICS  
EXERCISE SHEET 5, 29.10.2007**

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In the following,  $\mathfrak{f}$  and  $\mathfrak{p}$  stand for the semantic interpretations of signature symbols  $f$  and  $p$ , respectively, in the currently assigned interpretation.

**Problem 1.** Find a formula  $F$  and an interpretation  $I$  such that  $F$  is true for all valuations except for one.

**Problem 2.** Let  $F$  be the formula  $(\forall X)(p(X) \rightarrow p(f(X)))$ .

- (1) Let  $I$  be the interpretation having the set of natural numbers  $\mathbb{N}$  as domain and such that  $\mathfrak{p}(x) = \text{true}$  (or, if you prefer  $x \in \mathfrak{p}$ ) iff  $x$  is an even natural number and  $\mathfrak{f}(x) = x + 4$  for all  $x \in \mathbb{N}$ . Is  $I$  a model of  $F$ ?
- (2) Let  $D = \{a, b, c\}$ , let  $I$  be an interpretation having domain  $D$  and such that  $\mathfrak{p}(a) = \text{true}$ ,  $\mathfrak{p}(b) = \text{true}$ ,  $\mathfrak{p}(c) = \text{false}$ . Define  $\mathfrak{f}$  in such a way that the formula  $F$  is valid in  $I$ .

**Problem 3.** Consider the following formulae:

$$\begin{aligned} F_1 &= (\forall X)(\neg p(X, X)), \\ F_2 &= (\forall X)(\exists Y)p(X, Y), \\ F_3 &= (\forall X)(\forall Y)(\forall Z)(p(X, Y) \wedge p(Y, Z) \rightarrow p(X, Z)). \end{aligned}$$

- (1) Verify if the formulae above are simultaneously valid in the interpretation  $I$  having domain the set of natural numbers  $\mathbb{N}$  and mapping the symbol  $p$  as follows:  $\mathfrak{p}(x, y) = \text{true}$  iff  $x > y$ .
- (2) Verify if the formulae above are simultaneously valid in the interpretation  $I$  having domain the set of natural numbers  $\mathbb{N}$  and mapping the symbol  $p$  as follows:  $\mathfrak{p}(x, y) = \text{true}$  iff  $x \leq y$ .

**Problem 4.** The set  $\mathbb{F} = \{n_0, n_1, n_2, \dots\}$  of Fibonacci numbers is such that  $n_0 = 0$ ,  $n_1 = 1$  and  $n_{i+2} = n_{i+1} + n_i$ , for  $i \geq 0$ . Consider an interpretation  $I$  whose domain is the set  $\mathbb{N}$  of natural numbers and such that the unary predicate symbol  $q$  is interpreted by  $\mathbb{F}$ . Write a formula  $F$  such that  $I$  is a model for

$$(\forall X)(q(X) \leftrightarrow F).$$

Explain in detail the semantics of every symbol you use in  $F$ .

\* \* \*

The web page for the course is at [1]. You can find other exercises in Dan Richardson's notes, available from the web page.

References

1. Alessio Guglielmi, *CM20019—Computation III: Formal logic and semantics*, <http://cs.bath.ac.uk/ag/CM20019>, 2007.