

**CM20019—COMPUTATION III:
FORMAL LOGIC AND SEMANTICS
EXERCISE SHEET 5, 29.10.2007**

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In the following, \mathfrak{f} and \mathfrak{p} stand for the semantic interpretations of signature symbols f and p , respectively, in the currently assigned interpretation.

Problem 1. Find a formula F and an interpretation I such that F is true for all valuations except for one.

Problem 2. Let F be the formula $(\forall X)(p(X) \rightarrow p(f(X)))$.

- (1) Let I be the interpretation having the set of natural numbers \mathbb{N} as domain and such that $\mathfrak{p}(x) = \text{true}$ (or, if you prefer $x \in \mathfrak{p}$) iff x is an even natural number and $\mathfrak{f}(x) = x + 4$ for all $x \in \mathbb{N}$. Is I a model of F ?
- (2) Let $D = \{a, b, c\}$, let I be an interpretation having domain D and such that $\mathfrak{p}(a) = \text{true}$, $\mathfrak{p}(b) = \text{true}$, $\mathfrak{p}(c) = \text{false}$. Define \mathfrak{f} in such a way that the formula F is valid in I .

Problem 3. Consider the following formulae:

$$\begin{aligned} F_1 &= (\forall X)(\neg p(X, X)), \\ F_2 &= (\forall X)(\exists Y)p(X, Y), \\ F_3 &= (\forall X)(\forall Y)(\forall Z)(p(X, Y) \wedge p(Y, Z) \rightarrow p(X, Z)). \end{aligned}$$

- (1) Verify if the formulae above are simultaneously valid in the interpretation I having domain the set of natural numbers \mathbb{N} and mapping the symbol p as follows: $\mathfrak{p}(x, y) = \text{true}$ iff $x > y$.
- (2) Verify if the formulae above are simultaneously valid in the interpretation I having domain the set of natural numbers \mathbb{N} and mapping the symbol p as follows: $\mathfrak{p}(x, y) = \text{true}$ iff $x \leq y$.

Problem 4. The set $\mathbb{F} = \{n_0, n_1, n_2, \dots\}$ of Fibonacci numbers is such that $n_0 = 0$, $n_1 = 1$ and $n_{i+2} = n_{i+1} + n_i$, for $i \geq 0$. Consider an interpretation I whose domain is the set \mathbb{N} of natural numbers and such that the unary predicate symbol q is interpreted by \mathbb{F} . Write a formula F such that I is a model for

$$(\forall X)(q(X) \leftrightarrow F).$$

Explain in detail the semantics of every symbol you use in F .

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The web page for the course is at [1]. You can find other exercises in Dan Richardson's notes, available from the web page.

References

1. Alessio Guglielmi, *CM20019—Computation III: Formal logic and semantics*, <http://cs.bath.ac.uk/ag/CM20019>, 2007.