

**CM20019—COMPUTATION III:  
FORMAL LOGIC AND SEMANTICS  
EXERCISE SHEET 4, 25.10.2007**

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**Problem 1.** Find two terms  $s$  and  $t$  such that  $s$  is an instance of  $t$  but where  $s$  and  $t$  cannot be unified. (Note: we say that  $s$  is an *instance* of  $t$  if there exists a substitution  $\sigma$  such that  $s = t\sigma$ .)

**Problem 2.** Apply the unification algorithm to the following pairs of terms (variables are uppercase):

- (1)  $f(X, Y, Z)$  and  $f(a, Z, h(a))$ ;
- (2)  $f(g(X), g(c), Y)$  and  $f(g(g(Y)), X, a)$ ;
- (3)  $f(g(X), g(Z), Y)$  and  $f(g(g(Y)), g(X), Z)$ ;
- (4)  $f(g(X), Y, h(Z))$  and  $f(U, W, U)$ ;
- (5)  $f(a, X, g(h(Y)))$  and  $f(Z, i(Z, W), g(W))$ ;
- (6)  $f(X, X, Y)$  and  $f(g(Y), g(g(Z)), g(a))$ .

**Problem 3.** Which of the following sentences expresses the correct negation of ‘At night all cats seem to be black’?

- (1) At night no cat seems to be black.
- (2) Some nights there’s at least a cat that doesn’t seem to be black.
- (3) Some nights cats don’t seem to be black.
- (4) During the morning cats don’t seem to be black.
- (5) At night we can’t see cats.

**Problem 4.** Express the induction principle with a first order formula. (Note: the induction principle says that if a property is true for 0 and, whenever it is true for a natural number, it is true for its successor, then it is true for every natural number.)

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The web page for the course is at [1]. You can find other exercises in Dan Richardson’s notes, available from the web page.

References

1. Alessio Guglielmi, *CM20019—Computation III: Formal logic and semantics*, <http://cs.bath.ac.uk/ag/CM20019>, 2007.