

**CM20019—COMPUTATION III:
FORMAL LOGIC AND SEMANTICS
EXERCISE SHEET 1, 5.10.2007**

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Problem 1. Let us consider the language P of strings over the alphabet $\{(,)\}$; for example, $s = (()())()()$ (and $t = ()()$) are two strings in P of length, respectively, 12 and 6; of course, the empty string ϵ of length 0 belongs to P as well. A *prefix* of a string x is any string y such that there exists a string z for which $x = yz$; for example ϵ , $()$ (and $()()()$) are three of the thirteen prefixes of string s .

We now consider two notions defined for the strings of P :

- (1) A string x is *1-balanced* iff:
 - (a) x has an equal number of '('s and ')'s, and
 - (b) any prefix of x has at least as many '('s as ')'s.
- (2) *2-balanced* strings are such that:
 - (a) ϵ is 2-balanced;
 - (b) if x is 2-balanced then (x) is 2-balanced;
 - (c) if x and y are 2-balanced, then so is xy ;
 - (d) nothing else is a 2-balanced string.

Prove by induction on the length of strings that the two notions of 1-balanced string and 2-balanced string are equivalent, *i.e.*, prove that if x is 1-balanced then x is 2-balanced and vice versa.

Problem 2. We say that a binary relation R on a set S (*i.e.*, $R \subseteq S^2$) is

- *reflexive* if aRa for all $a \in S$;
- *symmetric* if aRb implies bRa ;
- *transitive* if aRb and bRc implies aRc .

(Note: we write, as customary, aRb instead of $(a, b) \in R$.)

Give an example of a relation that is symmetric and transitive but not reflexive.

Problem 3. Let \mathcal{P} be a set of properties of relation R ; for example, we can refer to Problem 2 and consider $\mathcal{P} = \{\text{reflexive, symmetric, transitive}\}$ and any of its subsets. We call the \mathcal{P} -closure of R the smallest relation R' such that $R \subseteq R'$ and such that R' possesses the properties in \mathcal{P} .

Find the {transitive}-closure, the {reflexive, transitive}-closure and the {symmetric}-closure of the relation

$$\{(1, 2), (2, 3), (3, 4), (5, 4)\} \quad .$$

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You can find these and more exercises in [2]. The web page for the course is at [1].

References

1. Alessio Guglielmi, *CM20019—Computation III: Formal logic and semantics*, <http://cs.bath.ac.uk/ag/cm20019>, 2007.
2. John E. Hopcroft and Jeffrey D. Ullman, *Introduction to automata theory, languages and computation*, Addison Wesley, 1979.