CM20019—COMPUTATION III: FORMAL LOGIC AND SEMANTICS EXERCISE SHEET 1, 5.10.2007

ALESSIO GUGLIELMI

Problem 1. Let us consider the language P of strings over the alphabet $\{(,)\}$; for example, $s = (()()))()()($ and $t = (()))()$ are two strings in P of *length*, respectively, 12 and 6; of course, the empty string ϵ of length 0 belongs to P as well. A *prefix* of a string x is any string y such that there exists a string z for which $x = yz$; for example ϵ, $(()($ and $(()()))()$ are three of the thirteen prefixes of string s.

We now consider two notions defined for the strings of P:

(1) A string x is *1-balanced* iff:
   (a) x has an equal number of ('s and)'s, and
   (b) any prefix of x has at least as many ('s as)'s.
(2) *2-balanced* strings are such that:
   (a) ϵ is 2-balanced;
   (b) if x is 2-balanced then (x) is 2-balanced;
   (c) if x and y are 2-balanced, then so is xy;
   (d) nothing else is a 2-balanced string.

Prove by induction on the length of strings that the two notions of 1-balanced string and 2-balanced string are equivalent, *i.e.*, prove that if x is 1-balanced than x is 2-balanced and vice versa.

Problem 2. We say that a binary relation R on a set S (*i.e.*, $R \subseteq S^2$) is

- *reflexive* if aRa for all $a \in S$;
- *symmetric* if aRb implies bRa;
- *transitive* if aRb and bRc implies aRc.

(Note: we write, as customary, aRb instead of $(a, b) \in R$.)

Give an example of a relation that is symmetric and transitive but not reflexive.

Problem 3. Let $\mathcal{P}$ be a set of properties of relation R; for example, we can refer to Problem 2 and consider $\mathcal{P} = \{\text{reflexive}, \text{symmetric}, \text{transitive}\}$ and any of its subsets. We call the $\mathcal{P}$*-closure* of R the smallest relation R' such that $R \subseteq R'$ and such that R' possesses the properties in $\mathcal{P}$.

Find the $\{\text{transitive}\}$-closure, the $\{\text{reflexive}, \text{transitive}\}$-closure and the $\{\text{symmetric}\}$-closure of the relation

$$\{(1, 2), (2, 3), (3, 4), (5, 4)\} \quad .$$

⋆ ⋆ ⋆

You can find these and more exercises in [2]. The web page for the course is at [1].

References

1. Alessio Guglielmi, *CM20019—Computation III: Formal logic and semantics*, `http://cs.bath.ac.uk/ag/CM20019`, 2007.
2. John E. Hopcroft and Jeffrey D. Ullman, *Introduction to automata theory, languages and computation*, Addison Wesley, 1979.