CM20019—COMPUTATION III: FORMAL LOGIC AND SEMANTICS COURSEWORK, DUE 22.11.2007

ALESSIO GUGLIELMI

Information and instructions for the coursework submission process:

- This coursework is to be submitted through the coursework post box located outside 1W2.23 by 22 November 2007 at 17:00. Please be careful to select the box relative to this course.
- This coursework is to be submitted in written form, on paper, bound securely. Please make sure it contains information to identify you, clearly written: surname, first name, and student number. You must include the coursework signature sheet.
- This coursework will contribute 25% of the final grade for this unit.
- This is an individual coursework, and it should be completed within the student's own time.
- Feedback will be provided by example solutions.

Plagiarism will be detected and will be penalised. Remember that, while there are not many ways to answer a problem correctly, there are infinitely many, **unique** ways of making mistakes. Remember also that **even the best make mistakes**.

⋆ ⋆ ⋆

Problem 1. Use the unification algorithm on the following pairs of terms and either find most general unifiers or tell when the terms are not unifiable and why.

(a) $f(X, Y)$ and $f(Z, X)$; [2 points]
(b) $f(X, Y)$ and $g(Z, X)$; [2 points]
(c) $f(X, X)$ and $f(Y, g(Y))$; [3 points]
(d) $f(X, Y, g(g(Z)))$ and $f(g(Y), g(Z), X)$; [4 points]
(e) $f(X, g(a))$ and $f(Y, X)$. [4 points]

Problem 2. Either prove the validity, by resorting to relevant definitions, or disprove it, by providing a counter-model, of the following formulae:

(a) $F = (\forall X)(\exists Y)(p(X) \to p(Y))$; [6 points]
(b) $G = (\exists X)(\forall Y)(p(X) \to p(Y))$. [9 points]

Then, answer the question:

(c) Are F and G equivalent? [5 points]

(*Note*: a counter-model for a formula is an interpretation in which the formula is false; two formulae are equivalent if whenever one is true in an interpretation, the other is true in that interpretation as well.)

Problem 3. We are given the predicate symbols p and q, and the function symbol f, all of which have arity 1. Let I be an interpretation such that:

- its domain is the set $\mathbb{N}$ of natural numbers (including 0);
- p is interpreted as $\{\, 3n \mid n \in \mathbb{N} \,\} \subset \mathbb{N}$;
- q is interpreted as $\{\, n \mid n \text{ is prime} \,\} \subset \mathbb{N}$;
- f is interpreted as the function $\mathbf{f}\colon \mathbb{N} \to \mathbb{N}$ such that $\mathbf{f}(n) = n + 6$.

Please answer the following questions in concise, not necessarily formal, but precise and unambiguous mathematical terms.

(a) Is the formula $p(f(X)) \wedge q(X)$ false in I for some valuation? If so, which one? If not, why not? [2 points]

(b) Is the formula $p(f(X)) \wedge q(X)$ true in I for some valuation? If so, which one? If not, why not? [2 points]

(c) Is the formula $\neg(\exists X)(p(f(X)) \wedge q(X))$ true in I? Explain. [2 points]

(d) Is the formula $(\forall X)(q(X) \to (p(f(X)) \vee q(f(X))))$ true in I? Explain. [5 points]

(e) Is the formula $(\forall X)(q(X) \to (p(f(X)) \vee q(f(X))))$ valid? Explain. [4 points]

(f) Find, in the given language, a satisfiable formula F such that it contains the symbol f and such that I is a model for

$$(\forall X)(F \to (p(X) \wedge q(X))) \quad .$$

[11 points]

(g) Find, in the given language, a formula F such that it contains the symbol f and such that

$$(\forall X)(F \to (p(X) \wedge q(X)))$$

is valid. [9 points]

(*Note*: a formula F is satisfiable if there is an interpretation and a valuation for which F is true; the set of prime numbers is $\{2, 3, 5, 7, 11, 13, 17, \dots\}$.)

Problem 4. Consider the following Prolog program:

```
a(0,X,X).
a(s(X),Y,s(Z)) :- a(X,Y,Z).

b(0,0).
b(s(0),s(0)).
b(s(s(0)),s(0)).
b(s(s(s(Y))),X) :- b(Y,Z),
                   b(s(Y),W),
                   b(s(s(Y)),U),
                   a(Z,W,V),
                   a(V,U,X).
c(X) :- b(Y,X).

d(X) :- b(X,X).
```

In your answers, please abbreviate any string of the kind `s(···s(0)···)` with the numeral corresponding to the number of its `s` symbols; for example, please abbreviate `s(s(s(0)))` as 3. Please explain the meaning of predicates in concise, not necessarily formal, but precise and unambiguous mathematical terms.

(a) What is the meaning of the predicate `a(X,Y,Z)`? [6 points]

(b) What is the meaning of the predicate `b(X,Y)`? [16 points]

(c) What is the meaning of the predicate `c(X)`? [4 points]

(d) What is the meaning of the predicate `d(X)`? [4 points]